An Estimation Method Matched to Microcomputer-Aided On-Line Measurement of $L_{eq}$ by Use of Statistical Information on the Noise Level Fluctuation

Yasuo Mitani¹, Noboru Nakasako², and Kazuhiro Tsutsumoto³

¹ Faculty of Engineering, Fukuyama University, Hiroshima, 729-0292 Japan
mitani@fuee.fukuyama-u.ac.jp

² School of Biology-Oriented Science and Technology, Kinki University, Wakayama, 649-6493 Japan
nakasako@info.waka.kindai.ac.jp

³ Faculty of Human Cultures and Sciences, Fukuyama University, Hiroshima, 729-0292 Japan
tsutsu@fucc.fukuyama-u.ac.jp

Abstract. In this paper, a general estimation method of $L_{eq}$ noise evaluation index is proposed using the statistical information on the noise level fluctuation. This method is given in an expansion type estimation formula universally applicable to the arbitrary non-Gaussian distribution, including a well-known simplified expression derived under the assumption of a standard Gaussian distribution as the first approximation. At this time, for extracting the statistical information on the noise level fluctuation, we propose an iterative processing algorithm matched to the microcomputer-aided on-line measurement. Next, a selection method of the optimum order of this expansion type $L_{eq}$ estimation formula is proposed by introducing Akaike’s information criterion (AIC). Finally, the effectiveness of the proposed method has been confirmed experimentally by applying it to actual road traffic noise.

1 Introduction

As is well known, the noise evaluation index, $L_{eq}$, plays an important role in the field of noise evaluation and regulation problems. In order to evaluate this index, the usual measurement method is given according to the original definition. That is, the $L_{eq}$ noise evaluation index is defined as a constant noise level whose noise energy value is equal to an averaged energy of the noise level fluctuation over a total measurement time interval. Nowadays, in an actual measurement, the noise level fluctuation is very often measured in a quantized amplitude form at every discrete time period using a digital-type instrument [1]. In the case where these data are used to evaluate $L_{eq}$, the following fundamental problems still remain as seen from the viewpoint of signal processing:

(1) In order to evaluate $L_{eq}$, it is necessary to obtain many level data with a fairly fine sampling period, since the noise energy fluctuation after the anti-logarithmic transformation of the sampled level datum fluctuates with large excursions, as
compared with the original decibel-scaled level fluctuation.

(2) In principle, the mean value of the noise energy fluctuation should be given by the sample mean operation based on the noise energy data especially with an equally quantized energy amplitude. If we use the noise energy fluctuation after transforming the measured noise level fluctuation with an equally quantized level amplitude into the energy scale through the anti-logarithmic transformation, a calculation error of the energy mean will occur because of the above level quantization.

In relation to the above problems, according to the current measurement techniques, the sampling period may greatly influence the accuracy of the measurement result. Namely, a fine sampling period related to the time constant of the integration giving the noise level will generally give a good approximation to the results obtained with a true integration. Therefore, if we construct the measurement system with the use of a microcomputer according to current practice, a huge memory capacity is needed for a long-term measurement with such a fine sampling period. From the above practical point of view, it is very convenient to utilize explicitly the statistical information on the decibel-scaled level fluctuation itself with the aid of the theory for evaluating $L_{eq}$.

The above information is fairly stable and reliable based on the averaging operation supported by a large number of data. Thus, in order to extract the statistical information, the sampling period of the noise level fluctuation can be grosser than that of current practice mentioned above. At the same time, each order moment statistics can be successively obtained by introducing an iterative calculation process. Therefore, the problem with a huge memory capacity can be solved by using this procedure.

In this paper, a general estimation method of $L_{eq}$ noise evaluation index is proposed using the statistical information on the noise level fluctuation. This method is given in an expansion type estimation formula universally applicable to the arbitrary non-Gaussian distribution, including a well-known simplified expression derived under the assumption of a standard Gaussian distribution as the first approximation. At this time, for extracting the statistical information on the noise level fluctuation, we propose an iterative processing algorithm matched to the microcomputer-aided on-line measurement. Next, a selection method of the optimum order of this expansion type $L_{eq}$ estimation formula is proposed by introducing AIC [2]. Finally, the effectiveness of the proposed method has been confirmed experimentally by applying it to actual road traffic noise.

## 2 General Method for Estimating $L_{eq}$

Let us consider the noise level fluctuation $x$ of an arbitrary non-Gaussian distribution type. As is well known, the relationship between the noise level fluctuation $x$ and the noise energy fluctuation $E$ is given as follows:

$$x = 10 \log_{10} \frac{E}{E_0} = M \ln \frac{E}{E_0} \left( M = \frac{10}{\ln 10} \right).$$  \hspace{1cm} (1)
where $E_0$ is the reference noise energy usually taken as $10^{-12} \text{ (W/m}^2\text{)}$. Here, we introduce the moment generating function $M_\theta(x)$ with respect to the noise level fluctuation $x$, as follows:

$$M_\theta(x) = \langle \exp(\theta x) \rangle = \exp \left( \theta M \ln \frac{E}{E_0} \right), \quad (2)$$

where $\langle \ast \rangle$ denotes an averaging operation with respect to the random variable $. The mathematical relationship between the $n$-th order cumulant $\kappa_n (n=1,2,\cdots)$ with respect to $x$ and the moment generating function $M_\theta(x)$ is given by

$$M_\theta(x) = \exp \left( \sum_{n=0}^{\infty} \frac{\kappa_n}{n!} \theta^n \right). \quad (3)$$

By replacing the parameter $\theta$ to $1/M$ in Eqs.(2) and (3), the mean value of $E$ can be easily obtained as follows:

$$\langle E \rangle = E_0 \exp \left( \sum_{n=0}^{\infty} \frac{\kappa_n}{n!} \frac{1}{M^n} \right). \quad (4)$$

Thus, a substitution of Eq.(4) into the definition of $L_{eq}$ yields a general expansion type expression for estimating $L_{eq}$, as follows [3]:

$$L_{eq} = 10 \log_{10} \frac{\langle E \rangle}{E_0} = \kappa_1 + \frac{\kappa_2}{2M} + \frac{\kappa_3}{6M^2} + \frac{\kappa_4}{24M^3} + \frac{\kappa_5}{120M^4} + \cdots$$

$$= \mu + 0.115 \sigma^2 + 8.84 \times 10^{-3} \kappa_3 + 5.09 \times 10^{-4} \kappa_4 + 2.34 \times 10^{-5} \kappa_5 + \cdots, \quad (5)$$

where $\mu (=\kappa_1)$ and $\sigma^2 (=\kappa_2)$ denote the mean value and the variance of $x$. From Eq.(5), it is possible to estimate $L_{eq}$ generally by reflecting not only lower order cumulants but also higher order cumulants in a hierarchical form. It should be noted that the above estimation formula agrees completely with a well-known simplified estimation formula [4] derived under the assumption of a standard Gaussian distribution as the first approximation:

$$L_{eq} = \mu + 0.115 \sigma^2, \quad (6)$$

since higher order cumulants $\kappa_n (n=3,4,\cdots)$ become zero for this special case. Thus, this estimation method shows a generalized form including the well-known
simplified estimation method as a special case.

To establish an estimation method of \( L_{eq} \) with the aid of a microcomputer, we must obtain the cumulant \( \kappa_n \) \((n = 1, 2, \cdots)\) by spending a little amount of its memory. To achieve this purpose, we can first calculate the \( m \)-th order moment of \( x \), especially by means of the following iterative process:

\[
\langle x^m \rangle_N = \frac{N-1}{N} \langle x^m \rangle_{N-1} + \frac{1}{N} x_N^m.
\]  

(7)

From Eq.(7), we can obtain the \( m \)-th order moment \( \langle x^m \rangle_N \) at the \( N \)-th measurement time based on the memorized past value of the \( m \)-th order moment \( \langle x^m \rangle_{N-1} \) at the \((N - 1)\)-th measurement time and the present datum \( x_N \) at the \( N \)-th measurement time, in an iterative form. After obtaining the \( m \)-th order moment within the specific measurement time interval using the above procedure, the resultant moment \( \langle x^n \rangle_{(n = 1, 2, \cdots, m)} \), can be transformed into the cumulant \( \kappa_n \) \((n = 1, 2, \cdots, m)\), as follows [5]:

\[
\begin{align*}
\kappa_1 &= \langle x \rangle, \\
\kappa_2 &= \langle x^2 \rangle - \langle x \rangle \kappa_1, \\
\kappa_3 &= \langle x^3 \rangle - 2 \langle x \rangle \kappa_2 - \langle x^2 \rangle \kappa_1, \\
\kappa_4 &= \langle x^4 \rangle - 3 \langle x \rangle \kappa_3 - 3 \langle x^2 \rangle \kappa_2 - \langle x^3 \rangle \kappa_1, \\
\kappa_5 &= \langle x^5 \rangle - 4 \langle x \rangle \kappa_4 - 6 \langle x^2 \rangle \kappa_3 - 4 \langle x^3 \rangle \kappa_2 - \langle x^4 \rangle \kappa_1, \cdots.
\end{align*}
\]  

(8)

Thus, we can estimate the objective \( L_{eq} \) by substituting the calculated values of cumulant statistics into Eq.(5).

3 Selection Method for the Optimum Order of Expansion Term

Here, it should be noticed that the sample number of actually obtained data is finite. Moreover, since it is impossible to calculate the infinite number of expansion terms in the actual data processing, the finite number of expansion terms must be inevitably employed. Therefore, we must establish a reasonable method for selecting the number of higher order correction terms in Eq.(5). From the practical point of view, we regard the remaining error after employing an appropriate expansion term as the meaningless error information. Therefore, in a case when the finite number of correction terms is employed, it is possible to employ AIC as an evaluation criterion for determining the optimum order of expansion expression. In this case, each expansion coefficient of higher order cumulants in Eq.(5) should be estimated in
advance by using least squares method, as follows:

\[ L_{eq} = \mu + 0.115\sigma^2 + \hat{a}\kappa_5 + \hat{b}\kappa_4 + \hat{c}\kappa_3 + \hat{d}\kappa_2 + \cdots. \]

(9)

Thus, we can obtain the optimum order so as to minimize the value of AIC [2], as follows:

\[ \text{AIC} = N\ln\hat{\sigma}_e^2 + 2\times(\text{number of correction terms}) \rightarrow \text{min}. \]

(10)

Here, \( N \) denotes the number of data and \( \hat{\sigma}_e^2 \) denotes the variance of estimation errors between the true values measured actually and the estimated values by using the estimation formula with the arbitrary order.

4 Experimental Work

For the purpose of confirming the effectiveness of the proposed estimation method, a measurement system has been constructed using a digital sound level meter and a portable microcomputer. The road traffic noise data of 100 kinds have been measured at various observation points in Fukuyama City by use of this measurement system. The measurement interval and its sampling period were selected to be 10 minutes and 0.2 seconds, respectively.

Table 1 shows the estimated values of expansion coefficients by use of least squares method. We calculated the variances of estimation errors between the true values measured actually and the estimated values by using the estimation formulae with finite expansion terms with the above estimated coefficients. The estimated values of AIC versus the number of correction terms for the measured road traffic noise are shown in Fig. 1. From this figure, the optimum order is 4 (i.e., the optimum number of correction terms is 2). The optimum estimation formula of \( L_{eq} \) is obtained as follows:

\[ L_{eq} = \mu + 0.115\sigma^2 + 8.83\times10^{-3}\kappa_3 + 5.08\times10^{-4}\kappa_4. \]

(11)

Table 1. The estimated values of expansion coefficients by use of least squares method

<table>
<thead>
<tr>
<th>Employment of correction terms up to</th>
<th>Estimated expansion coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{a} )</td>
</tr>
<tr>
<td>1st term</td>
<td>1.09\times10^{-2}</td>
</tr>
<tr>
<td>2nd term</td>
<td>8.83\times10^{-3}</td>
</tr>
<tr>
<td>3rd term</td>
<td>1.46\times10^{-2}</td>
</tr>
<tr>
<td>4th term</td>
<td>1.00\times10^{-2}</td>
</tr>
</tbody>
</table>
An Estimation Method Matched to Microcomputer-Aided

It is noticeable that the coefficients in the above optimum estimation formula are approximately equal to the expansion coefficients given in Eq.(5). This is because of the reasonable consideration of non-Gaussian property based on the fairly stable information from the first order cumulant to fourth order cumulant.

We apply the proposed method to 3 kinds of data selected randomly in the above 100 kinds of measured data (these are defined as Case A, Case B and Case C). The estimated results of $L_{eq}$ by using the proposed estimation method are shown in Table 2. In order to confirm the practical effectiveness of the proposed method, it is applied to the other 3 kinds of data measured at the other observation points (these are defined as Case D, Case E and Case F). The estimated results for these cases are shown in Table 3. According to these tables, the estimated values by using the proposed method are in good agreement with the experimental values.

![Fig. 1. The values of AIC versus the number of correction terms for the measured road traffic noise data](image)

<table>
<thead>
<tr>
<th>Number of Correction Terms</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1000</td>
</tr>
<tr>
<td>3</td>
<td>-3000</td>
</tr>
<tr>
<td>4</td>
<td>-4000</td>
</tr>
</tbody>
</table>

Table 2. The estimated results for $L_{eq}$ by use of the proposed method

<table>
<thead>
<tr>
<th>Case</th>
<th>Experimental values (dB)</th>
<th>Estimated values (dB)</th>
<th>Estimation errors (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>72.8</td>
<td>72.7</td>
<td>-0.1</td>
</tr>
<tr>
<td>B</td>
<td>73.2</td>
<td>72.9</td>
<td>-0.3</td>
</tr>
<tr>
<td>C</td>
<td>74.4</td>
<td>74.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3. The estimated results for $L_{eq}$ by use of the proposed method

<table>
<thead>
<tr>
<th>Case</th>
<th>Experimental values (dB)</th>
<th>Estimated values (dB)</th>
<th>Estimation errors (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>81.3</td>
<td>81.7</td>
<td>0.4</td>
</tr>
<tr>
<td>E</td>
<td>81.6</td>
<td>81.6</td>
<td>0.0</td>
</tr>
<tr>
<td>F</td>
<td>75.7</td>
<td>75.4</td>
<td>-0.3</td>
</tr>
</tbody>
</table>
5 Conclusion

In this paper, a general method of estimating $L_{eq}$ noise evaluation index has been proposed using the statistical information on the noise level fluctuation. This method has been given in an expansion type estimation formula universally applicable to the arbitrary non-Gaussian distribution, including a well-known simplified expression derived under the assumption of a standard Gaussian distribution as the first approximation. At this time, for extracting the statistical information on the noise level fluctuation, we proposed an iterative processing algorithm matched to the microcomputer-aided on-line measurement. Next, a selection method of the optimum order of this expansion type $L_{eq}$ estimation formula has been proposed by introducing AIC. Finally, the effectiveness of the proposed method has been confirmed experimentally by applying it to actual road traffic noise.

Of course, this study is in its early stage and has been focused only on its fundamental aspects. Accordingly, there still remain future problems, as follows:

(1) This method must be applied to many other actual cases to broaden and confirm its further effectiveness.

(2) We must propose an estimation method for the arbitrary $L_x$ noise evaluation indices (e.g., $L_5$, $L_{10}$, $L_{50}$, $L_{90}$, $L_{95}$, ...).

Acknowledgements

The authors would like to express their cordial thanks to Dr. M. Ohta, Mr. M. Shigenawa and Mr. S. Shimizu for helpful discussions.

References


Biography

▲ Name: Yasuo Mitani
Address: Faculty of Engineering, Fukuyama University, Sanzo, Gakuen-cho 1, Fukuyama City, Hiroshima, 729-0292 Japan
Education & Work experience: He received the B.E. and M.E. degrees from Fukuyama University in 1979 and 1981, respectively. He received the Dr. Eng. degree from Hiroshima University in 1990. He is now a professor at Faculty of Engineering, Fukuyama University. His research interest includes acoustic signal processing, sound and vibration control, and their application to acoustics.
Tel: +81-84-936-2111
E-mail: mitani@fuee.fukuyama-u.ac.jp

▲ Name: Noboru Nakasako
Address: Faculty of Biology-Oriented Science and Technology, Kinki University, Nishi-Mitani 930, Uchita-cho, Naga-gun, Wakayama, 649-6493 Japan
Education & Work experience: He received the B.E., M.E., and Dr. Eng. degrees from Hiroshima University in 1982, 1984, and 1990, respectively. He is now a professor at Faculty of Biology-Oriented Science and Technology, Kinki University. His research interest includes acoustic signal processing, sound and vibration control, independent component analysis and their application to acoustics.
Tel: +81-736-77-3888
E-mail: nakasako@info.waka.kindai.ac.jp

▲ Name: Kazuhiro Tsutsumoto
Address: Faculty of Human Cultures and Sciences, Fukuyama University, Sanzo, Gakuen-cho 1, Fukuyama City, Hiroshima, 729-0292 Japan
Education & Work experience: He received the B.E. degree from Fukuyama University in 1979. He is now an associate professor at Faculty of Human Cultures and Sciences, Fukuyama University. His research interest includes acoustic signal processing, sound and vibration control, and their application to acoustics.
Tel: +81-84-936-2111
E-mail: tsutsu@fucc.fukuyama-u.ac.jp

ⓒGESTS-Oct.2005